Generating Logically Challenging Logic Puzzles

Using a two population genetic algorithm

Intro to Logic Grid Puzzles

Scenario

Mr. Body was found dead in the great hall, at 4:35PM. However police can confirm he was killed in a different part of the house between 1:00 PM and 4:00pm. Notes from his desk indicate he had plans to meet with 4 individuals, Ms. Scarlet, Mrs. White, Prof. Plum, and Col. Mustard all at different times and even in different rooms of the house. Additionally 4 potential murder weapons have been identified: a knife, a rope, a candlestick, and a wrench. Each weapon has a different set of fingerprints, indicating a different person held each item.

Can you figure out which suspect was in which room, at what time, with what weapon?

Example Puzzle 1

	Ms. Scarlet	Ms. White	Col Mustard	Prof. Plum	Knife	Rope	Candle	Wrench
1:00 PM								
2:00 PM								
3:00 PM								
4:00 PM								
Knife								
Rope								
Candle								
Wrench								

Clues

1. Either the suspect Prof. Plum or the weapon Rope is the hour 4:00 PM
2. The suspect Col. Mustard is 2 hours before the weapon Knife
3. The suspect Mrs. White is at least 1 hour before the weapon Wrench
4. The weapon Rope is the hour 3:00 PM

Solution

	Ms. Scarlet	Ms. White	Col Mustard	Prof. Plum	Knife	Rope	Candle	Wrench
1:00 PM	X	O	X	X	X	X	O	X
2:00 PM	X	X	O	X	X	X	X	O
3:00 PM	O	X	X	X	X	O	X	X
4:00 PM	X	X	X	O	O	X	X	X
Knife	X	X	X	O				
Rope	O	X	X	X				
Candle	X	O	X	X				
Wrench	X	X	O	X				

Clues

1. Either the suspect Prof. Plum or the weapon Rope is the hour 4:00 PM
2. The suspect Col. Mustard is 2 hours before the weapon Knife
3. The suspect Mrs. White is at least 1 hour before the weapon Wrench
4. The weapon Rope is the hour 3:00 PM

FI-2Pop

Genetic Algorithm

```
Genetic_Algorithm() {
                Initialize random population;
                Evaluate the population;
                Generation = 0;
     While termination criterion is not
     satisfied {
            Generation = Generation + 1;
            Select good chromosomes by
            reproduction procedure;
            Perform crossover with probability
            crossover (Pc);
            Perform mutation with probability
            of mutation (Pm);
            Evaluate the population;
     }
}
```

Genetic Operators

- Representation: how to represent a solution (individual)
- Initialization: a way to randomly generate a new individual
- Mutation:a way to randomly make a small change to an individual
- Cross-over: a way to combine two individuals together
- Fitness: a way to evaluate the quality of the individual

Constrained Environment

- How should you deal with infeasible individuals?
  - Death penalty: give infeasible individuals 0 fitness
    - Leads to loss of genetic information
  - Fitness penalty: reduce the fitness of infeasible individuals
    - Challenging to find right penalty

FI-2Pop

- A feasible-infeasible two population genetic algorithm
- Two populations are maintained

Infeasible

- Individuals that do not meet feasibility constraint
- Selected only to reach feasibility

Feasible

- Individuals that do meet feasibility constraint
- Selected for optimization criteria

PseudoCode

- Randomly generate n individuals
- Place individuals in feasible or infeasible population
- While resources last
  - Choose population based on current size
  - Select 2 individuals from population
  - Mutate and cross over individuals
  - Place children into feasible or infeasible population
    - If infeasible fitness is feasibility
    - If feasible fitness if optimization

Problem Space

Overview

- Input
  - N categories each with M entities
  - Categories can be categorical or numerical
- Output
  - A list of hints that can used to solve a problem

Hint Grammar

Type	Production Rules	example
is	The [cat1] [ent] is the [cat2] [ent]	The suspect Ms. Scarlet is the weapon Knife.
not	The [cat1] [ent] is not the [cat2] [ent]	The suspect Ms. Scarlet is not the weapon Knife.
before	The [alph] [ent1] is at least 1 [num] before the [alph] [ent2] The [alph] [ent1] is [int] [num]s before the [alph] [ent2]	The suspect Ms. Scarlet is 2 hours before the weapon Knife
or	Either the [cat1] [ent] or [cat2] [ent] is the [cat3] [ent] Either [cat1] [ent1] or [cat1] [ent2] is the [cat2] [ent]	Either the suspect Ms. Scarlet or the suspect Col. Mustard is the weapon Knife
complex or	Either [is] or [is]	Either the suspect Ms. Scarlet is the weapon Knife or the suspect Col. Mustard is the room Living Room

Table 1: The production rules for producing hints as discussed in subsection 3.2. Each line represents a different production rule.

Logical Meaning

- is: The two entities are linked
- not: The two entities are not linked
- before: The first entity is $k < (m - 2)$ units smaller than the
- second entity in the numerical category
- or: Either the first or the second entity is linked to the third
- entity, but not both
- complex or: Either the first is statement or the second is statement is true, but not both.

Solver

Algorithm 1: Solve Puzzle

Input : A list of hints H, and a puzzle P

create list l containing all hints in H
while *l is not empty, P was modified last loop, and no contradictions have been found* **do**
    create an empty list l_{temp}
    for *h in l* **do**
        attempt to update P using the logic of h
        if *h created a contradiction* **then**
            set P to be invalid
            break
        else
            if *P was changed* **then**
                update P based on deductive reasoning
            end if
            if *h is not completely satisfied by P* **then**
                add h to l_{temp}
            end if
    end for
    set l to l_{temp}
end while
return P

Genetic Operators

- Initialization
  - Randomly generate between 3 and 5 hints
- Mutation
  - Deletion: select a random hint to delete
  - Insertion: generate a new hint to add
- Cross-over
  - Combine hint list and randomly shuffle between children

Infeasible Fitness

- Infeasible individuals are those that are not solvable
  - Validity: logical inconsistencies
  - Completion: how much of the grid you can fill

$$0.33\left(\frac{f}{e+f}\right) + 0.33\left(\frac{c}{r}\right) + 0.33\left(\begin{cases} 0 & \text{if } v \geq 10 \\ 1-(v/10) & \text{otherwise} \end{cases}\right)$$

- f is the number of filled ("X" or "O") cells
- e is the number of empty cells
- c is the number of rows in a subgrid with exactly one "O"
- r is the total number of rows in all subgrids
- v is the number of logical violations

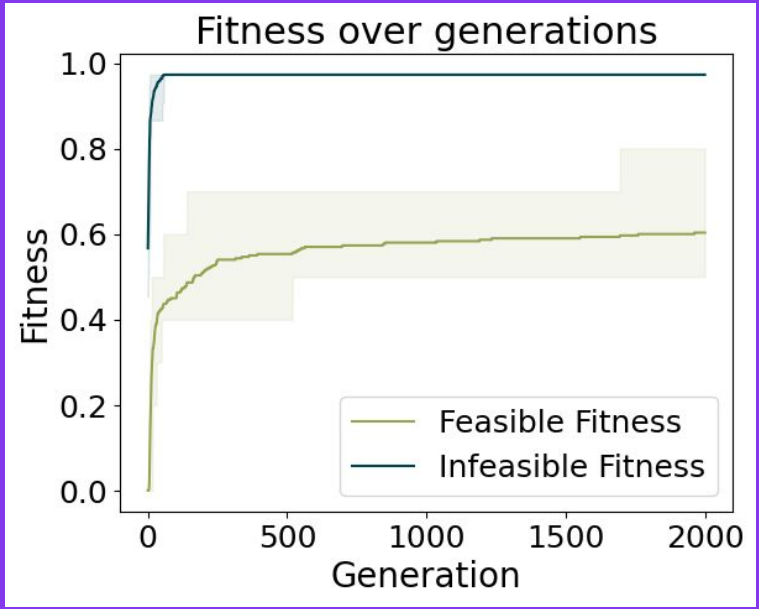
Feasible Fitness

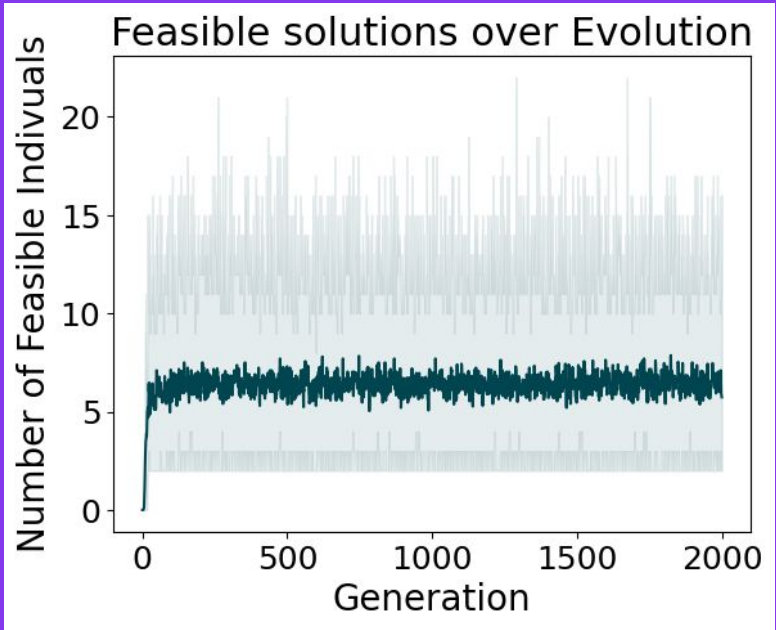
- Difficulty: the number of outer loops the solver takes to solve the puzzle
- Hint size: the number of hints

$$0.5(1 - \frac{h}{10}) + 0.5(\begin{cases} 1 & \text{if } l \geq 7 \\ l/7 & \text{otherwise} \end{cases})$$

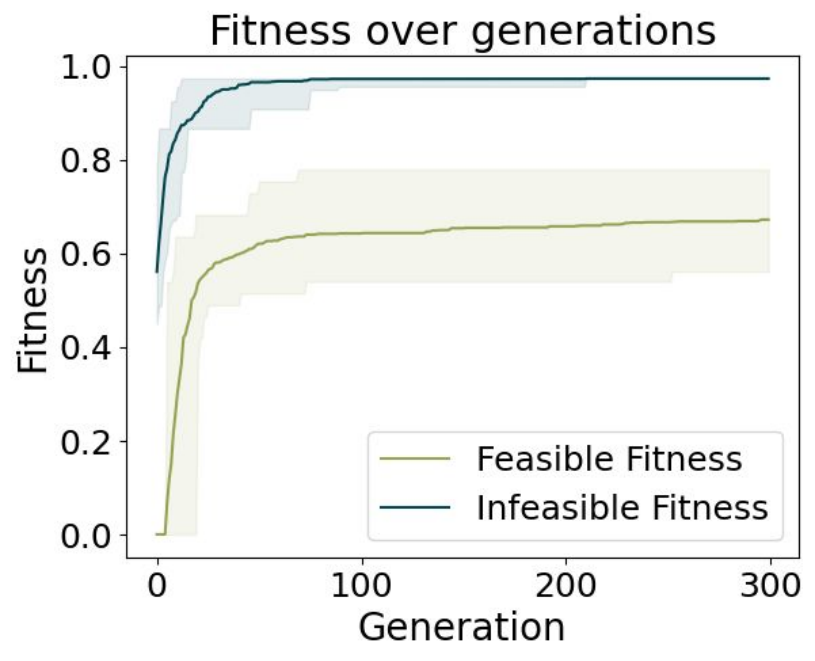
Results

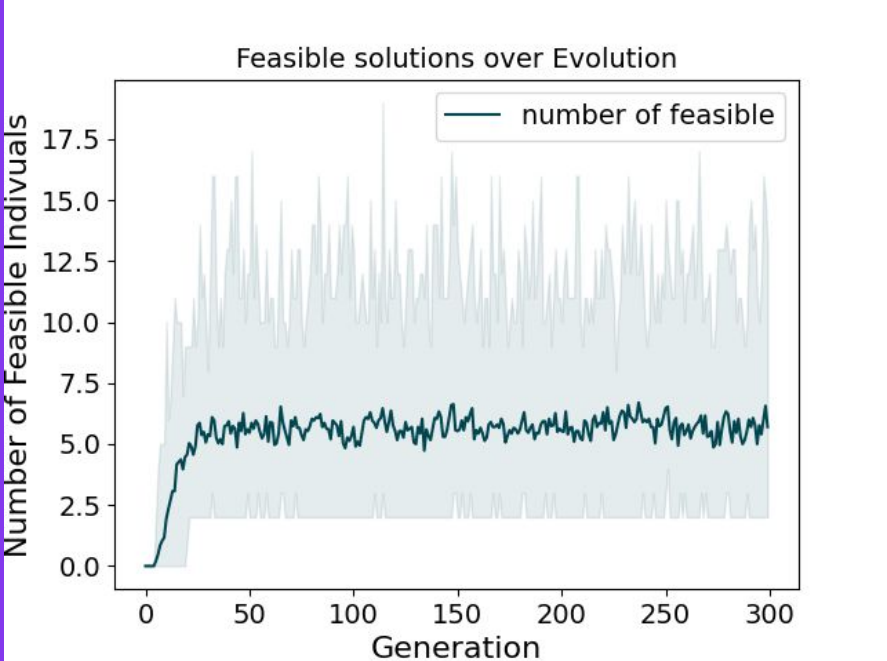
Difficulty only



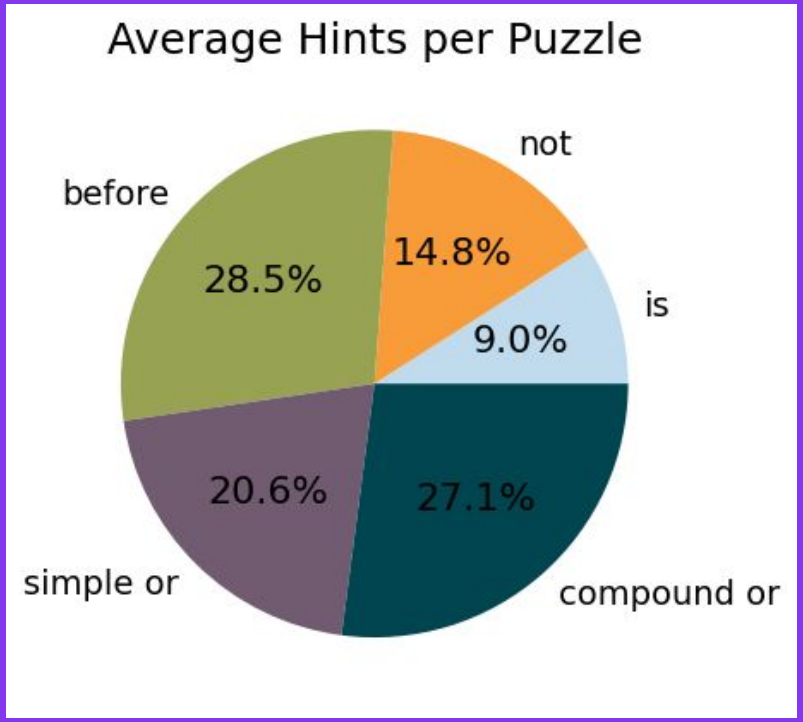


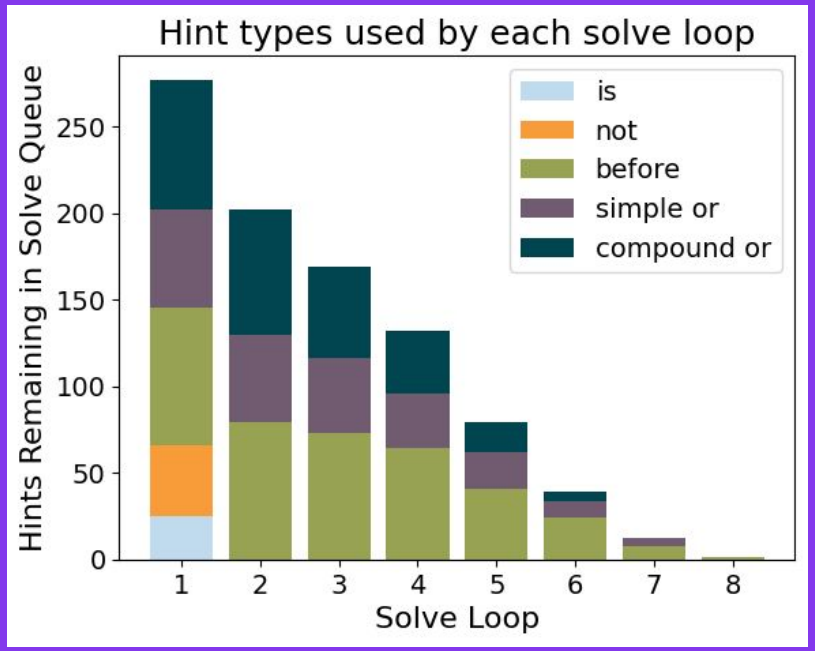
Hint size and Difficulty





Difficulty only (con)





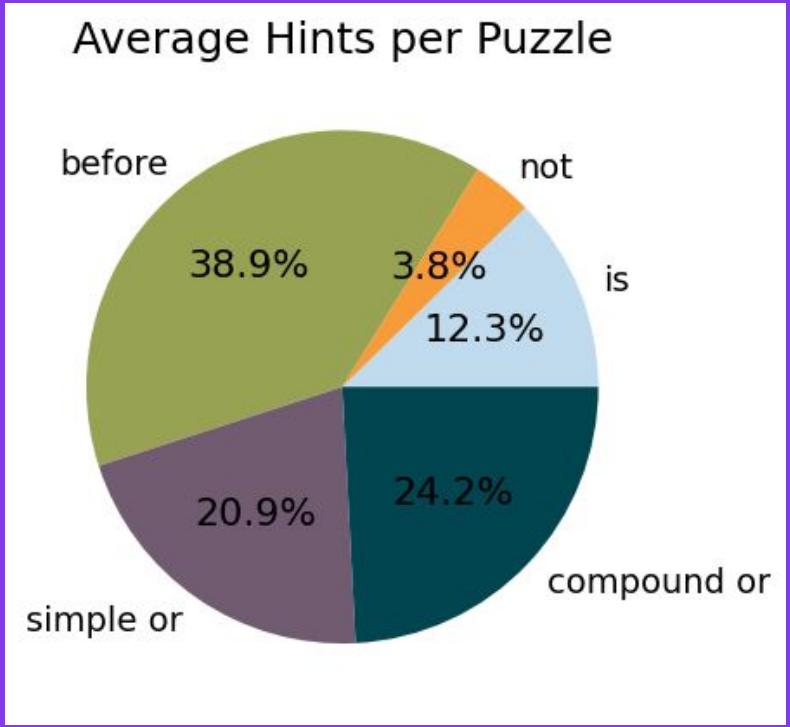
Example

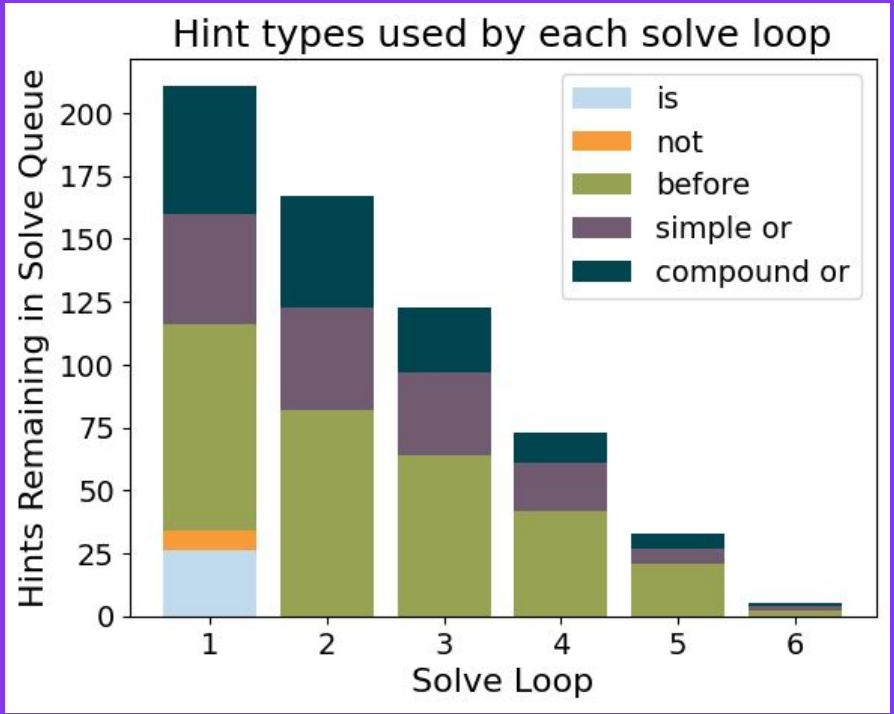
	Ms. Scarlet	Mrs. White	Col. Mustard	Prof. Plum	Knife	Rope	Candle Stick	Wrench	Ballroom	Living room	Kitchen	Study
1:00 PM												
2:00 PM												
3:00 PM												
4:00 PM												
Ball Room												
Living Room												
Kitchen												
Study												
Knife												
Rope												
Candle Stick												
Wrench												

Clues

1. Either the room Living Room or the suspect Prof. Plum is the weapon Rope
2. Either The room Living Room is the weapon Candle Stick or The room Study is the hour 3:00 PM
3. The suspect Col. Mustard is at least 1 hour before the room Living Room
4. Either The suspect Ms. Scarlet is the hour 3:00 PM or The hour 1:00 PM is the room Study
5. The suspect Col. Mustard is 1 hours before the room Ball room
6. Either The room Kitchen is the weapon Wrench or The weapon Rope is the hour 1:00 PM
7. The suspect Mrs. White is the hour 3:00 PM
8. Either The suspect Prof. Plum is the weapon Rope or The suspect Prof. Plum is the room Study
9. The weapon Knife is not the suspect in the category Prof. Plum
10. The room Ball room is 1 hours before the room Living Room

Hint size and Difficulty (con)





Example

	Ms. Scarlet	Mrs. White	Col. Mustard	Prof. Plum	Knife	Rope	Candle Stick	Wrench	Ballroom	Living room	Kitchen	Study
1:00 PM												
2:00 PM												
3:00 PM												
4:00 PM												
Ball Room												
Living Room												
Kitchen												
Study												
Knife												
Rope												
Candle Stick												
Wrench												

Clues

1. The suspect Prof. Plum is 1 hours before the suspect Col. Mustard

2. The suspect Ms. Scarlet is 1 hours before the weapon Wrench

3. The hour 2:00 PM is not the room in the category Ball room

4. The room Kitchen is at least 1 hour before the weapon Candle Stick

5. Either The hour 2:00 PM is the weapon Rope or The hour 1:00 PM is the room Kitchen

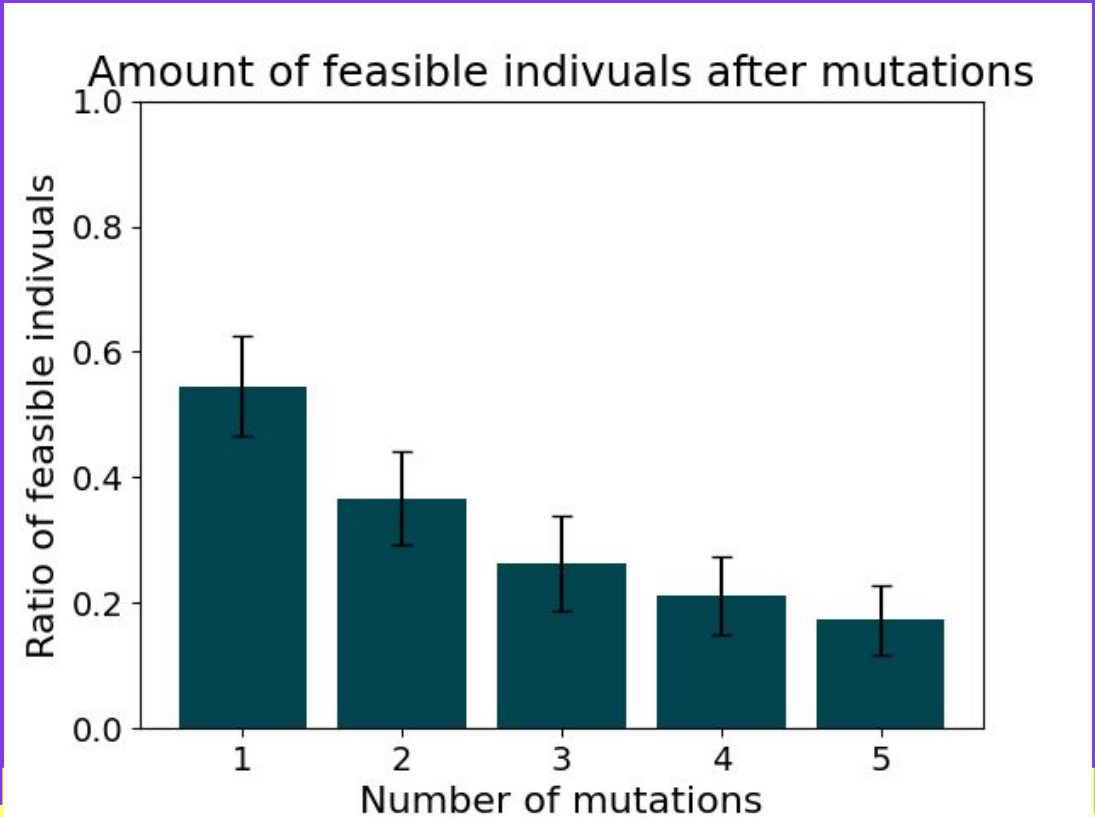
6. The room Study is 2 hours before the room Kitchen

Qualitative Analysis

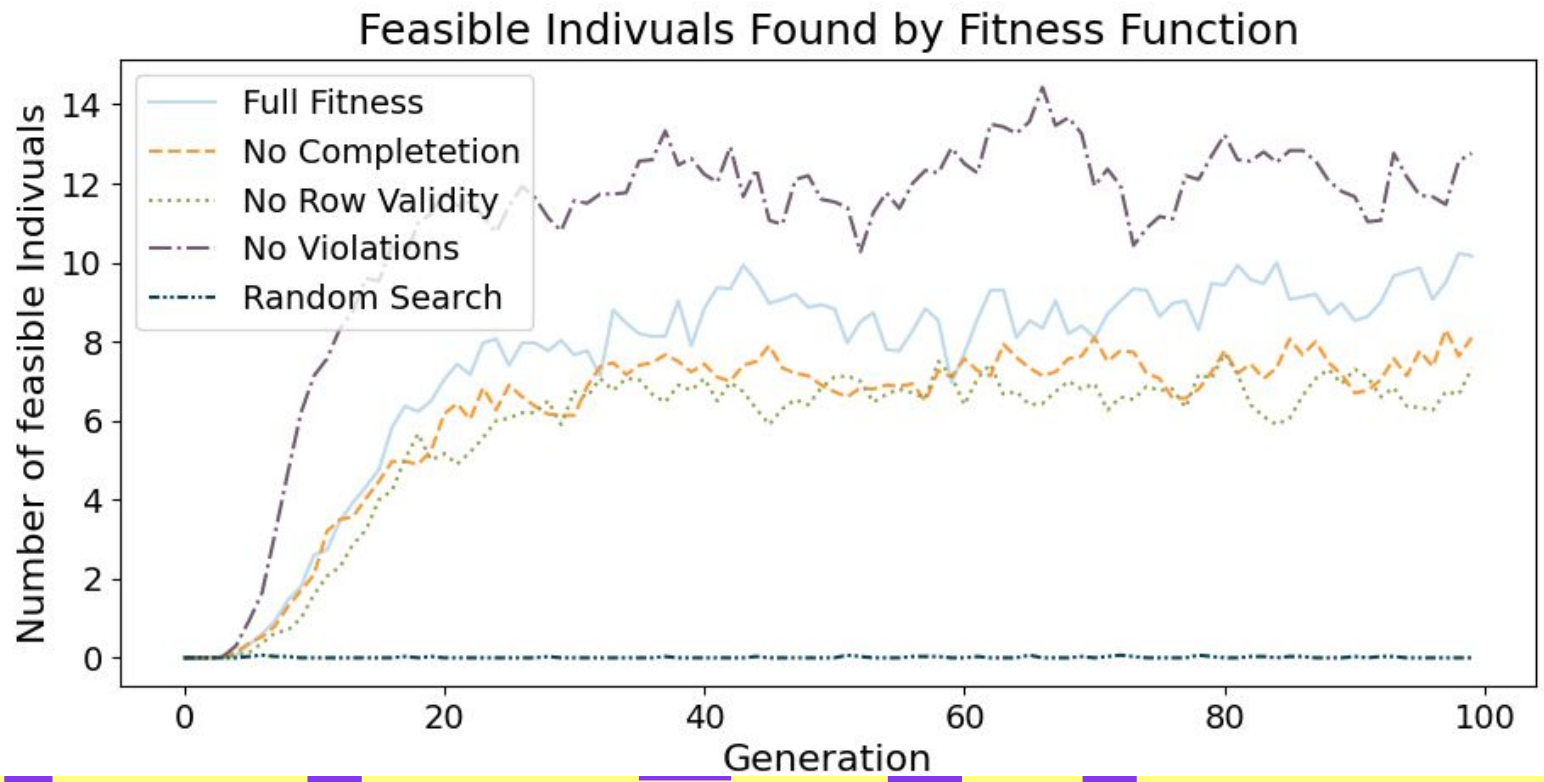
- Difficulty Only
  - Produces a large number of hints
  - Includes many redundancies
  - Includes several trivial deductions
- Hint size and difficulty
  - As challenging as difficulty only
  - Little redundancies
  - Appears similar to human authored puzzles

Other Experiments

Feasibility after Mutation



Infeasible Fitness



Conclusions

Conclusions

- First academic to generate this kind of puzzle
- Feasible individuals are found very quickly despite the highly constrained environment
- Puzzles presented sufficient challenge to the authors of this paper
- Selecting for hint sizes appears sufficient to reduce redundancies

Future Work

- Evolving a variety of difficulties - Map-Elites
- Other notations of difficulty
  - Types of deductions needed
- Combining hints with narrative